HW 6.4 SOL

February 5, 2018 1:53 PM

Name:_____ **Math 10/11 Enriched Section 6.4 Counting with Multiple Cases**

1. Andy and Brad are to sit anywhere in a row of 10 seats. How many ways can they be seated if they can not sit next to each other?

① All the ways they can sit: $10C_2 = \frac{10 \times 9}{1 \cdot 2} = 45$

② Ways to sit together: 1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-8, 8-9, 9-10 ⇒ $9 \times 2 = 18$

③ Complement (subtract): Total ways − what you don't want = $45 - 18 = 27$

2. There are eight males and five females applying to be in a team of four people. How many teams are possible if there must be at least one female?

① 8M, 5M ⇒ 13 ppl

# of ways to select 4 ⇒ $13C_4$

$\frac{13 \times 12 \times 11 \times 10}{1 \times 2 \times 3 \times 4} = 715$

② All teams with no females: $8C_4 = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$

③ Teams with at least one girl: $715 - 70 = 645$

3. How many five card hands are possible if there must be at least 3 hearts?

① At least 3 hearts: $3H2\bar{H} + 4H1\bar{H} + 5H$.

$13C_3 \times 39C_2 + 13C_4 \times 39C_1 + 13C_5$

$\frac{13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3} \times \frac{39 \cdot 38}{1 \cdot 2} + \frac{13 \cdot 12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3 \cdot 4} \times 39 + \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 45}$

=

4. How many five card hands are possible if there are 2 or more kings?

① $2k\,3\bar{k} + 3k \cdot 2\bar{k} + 4k \cdot 1\bar{k}$

$4C_2 \times 48C_3 + 4C_3 \cdot 48C_2 + 4C_4 \cdot 48C_1$

$6 \times \frac{48 \times 47 \times 46}{1 \cdot 2 \cdot 3} + 4 \times \frac{48 \times 47}{1 \times 2} + 1 \times 48$

= _____ + _____ + _____

5. How many different 4 letter words can be created using the letters from the word "MIKAYLA"

Since there are two A's, separate into 2 cases.

#1) Two A's: MIKYL (AA) $\quad \frac{4!}{2!} \times \frac{5 \times 4}{} = 120$

#2) 1 A or No A. $\quad \frac{6 \times 5 \times 4 \times 3}{} = 360$

Total $360 + 120 = 480$

6. In how many different ways can 3 men and 4 women be placed into two groups of two people and one group of three people if there must be at least one man and one woman in each group?

① Groups

_ _ _ , _ _ , _ _

First decide how many ways to get your first group: $3C_1 \times 4C_2$ [1W : 2M] $3 \times 6 = 18$

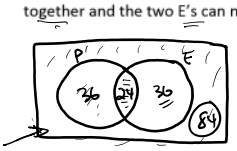
② Now decide how many ways these two groups can be made:

WA MA : WB MB or WA MB : WB MA

only two ways.

③ $18 \times 2 = 36$ ways.

7. How many distinct ways can the letters of the word PEOPLE be arranged so that the two P's can not be together and the two E's can not be together?



$\frac{6!}{2!2!} = 180$

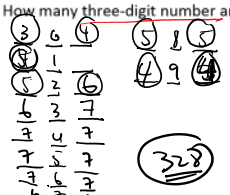
PEOPLE $\frac{5!}{2!} = 60$

PEOPL $\frac{5!}{2} = 60$

PEOL $4!$

$180 - 36 - 24 - 36 = 84$

8. How many three-digit number are there such that no two digits next to each other differ by more than 3?



328

0 1 2 3 4 5 6 7 8 9

0: 1, 2, 3 ~~4~~

9: 6, 7, 8, 9

1: ~~3~~, 1, 2, 3, 4

8: 5, 6, 7, 8, 9

2: ~~4~~, 1, 2, 3, 4, 5

7: 5, 6, 7, 8, 9

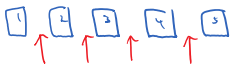
3: ~~4~~, ~~5~~, 1, 2, 3, 4, 5, 6

1

6

4: 1, 2, 3, 4, 5, 6, ~~7~~

9. There are five people waiting in a bank with two tellers. In how many ways can five people line up in the lines if there must be at least one person in each line?



$\Rightarrow 5! \times 4$

$= 480$

10. How many different four digit numbers can be obtained by using any four of the five digits: 2 ,3, 4, 4, and 4?

① WE HAVE 3 Four's → 2 CASES:

CASE 1) 3 Four's: $4 \times 2 = 8$

CASE 2) 2 Four's: $\frac{4!}{2!} \times 2 \times 1 = 12$

CASE 3) ONE 4 or No 4's: NONE

② Total $= 8 + 12 = 20$

11. The letters in the word HAWAII are each written on a card and four cards are randomly drawn in order to spell a word. How many different words can be created?

① WE HAVE 2 A's & 2 I's

CASE #1 2A's 2I's: $\frac{4!}{2!2!} = 3$

CASE #2) 2A's & 1 or No I's [AA, HWI]: $\frac{4!}{2} \times 3 \times 2 = 36$

CASE #3) 2 I's & 1 or No A's [II HWA]: $\frac{4!}{2} \times 3 \times 2 = 36$

CASE #4) No Repeats [HWAI]: $4! = 24$

② Total $= 36 + 36 + 3 + 24$

$= 72 + 27$

$= 99$

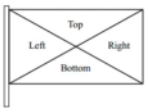
12. A school organization consists of 5 teachers, 7 parents and 6 students. A subcommittee of 5 people is to be created with at least 2 teachers. How many different subcommittees can be formed?

① AT LEAST 2 TEACHERS : $2T\,3\overline{T} + 3T\cdot 2\overline{T} + 4T\cdot 1\overline{T} + 5T$.

$(5C_2 \times 13C_3) + (5C_3 \times 13C_2) + (5C_4 \times 13C_1) + 5C_5$.

= _____ + _____ + _____ + _____

13. A rectangular flag is divided into four triangles, labelled Left, Right, Top, and Bottom, as shown. Each triangle is to be coloured one of red, white, blue, green , and purple so that no two triangles that share an edge are the same colour. How many different flags can be made? Pascal 2014



① Top 4 choices.

② L & R:  SAME colour (3 choices) or DIFF colour (2 choices)

③ Bottom: 3 choices / 2 choices

$4 \times 1 \times 3 = 12$ + $4 \times 1 \times 2 = 8$ = 20

14. How many four digit even numbers are there if none of the digits can be repeated?

① THE LAST DIGIT DICTATES THIS QUESTION

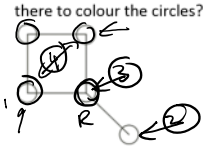
0, 2, 4, 6, 8.

CASE #1) $9 \times 8 \times 7 \times 1$ (1st: 1–9; last: zero)

CASE #2) $8 \times 8 \times 7 \times 4$ (1st: 1–9 (−1); last: 2, 4, 6, 8.) $\Rightarrow 9(56) + 32(56) - 41(56) = 2296$

2
56
41
56
224×
2296

15. Five circles are drawn on a piece of paper and connected as shown. Each circle must be coloured red, blue, or green. Two circles connected by a straight line may not be coloured the same. How many different ways are there to colour the circles? Pascal 2008



R: G—G → ③; G—B → ①; B—B → ③; B—G → ①

GG, BR: $3 \times 2 \times 2 \times 2 = 24$

GB, BG: $3 \times 2 \times 2 \times 1 = 12$

$= 36$

16. Starting with the 2, the number 2005 can be formed by moving either horizontally, vertically, or diagonally from square to square in the grid. How many different paths can be followed to form 2005? Pascal 2005

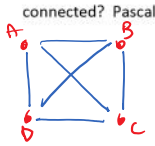
5	5	5	5	5
5	0	0	0	5
5	0	2	0	5
5	0	0	0	5
5	5	5	5	5

2 cases: (up/down/L/R) or (diagonals)

$1 \times 4 \times 3 + 1 \times 4 \times 5$

$12 + 20 = 32$

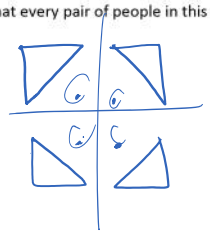
17. There are four people in a room. For two people, there is a 50% chance that they are friends. Two people are connected if: i) They are friends ii) A third person is friends with both of them or iii) they have different friends who are friends of each other. What is the probability that every pair of people in this room is connected? Pascal 2012:



Total ways = 2^6

0 lines	1
1 line	6
2 lines	$6C_2 = 15$
3 lines	4

24



Prob = $\frac{2^6 - 24}{2^6}$

$= \frac{64-24}{64}$

$= \frac{32}{64} = \frac{19}{32}$

18. Challenge: Six soccer teams are competing in a tournament in UBC. Every team is to play three games, each against a different team. (Note that not every pair of teams plays a game together) Bob is in charge of pairing up the teams to create a schedule of games that will be played. Ignoring the order and times of the games, how many different schedules are possible? Pascal 2010

① Count the number of ways teams don't play with each other

Case #1) Big circle

$\frac{6!}{5} \div 2 = 60$

clockwise / counter cw

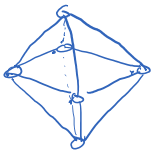
Case 2) 2 triangles

$\frac{6C_3}{2} = 10$

Total = 60 + 10

= 70

19. Challenge: Six ants simultaneous stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex? AMC 12



20. Challenge: Ten women sit in 10 seats in a line. All of the 10 get up and then reseat themselves using all 10 seats, each sitting in the seat she was in before or a seat next to the one she occupied before. In how many ways can the woman be reseated? Amc 12 2009

25. Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex?

(A) $\frac{5}{256}$ (B) $\frac{21}{1024}$ (C) $\frac{11}{512}$ (D) $\frac{23}{1024}$ (E) $\frac{3}{128}$

Problem 21

Ten women sit in 10 seats in a line. All of the 10 get up and then reseat themselves using all 10 seats, each sitting in the seat she was in before or a seat next to the one she occupied before. In how many ways can the women be reseated?

(A) 89 (B) 90 (C) 120 (D) 2^{10} (E) $2^2 3^5$

How many four-digit positive integers have at least one digit that is a 2 or a 3?

(A) 2439 (B) 4096 (C) 4903 (D) 4904 (E) 5416